Integrated quality and maintenance decision in a production inventory model with multiple market demand

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Abstract: In the proposed model, the supplier has the difference in selling season in between geographically dispersed market for the importance of firm profitability. Most research paper has been examined the deterioration of the production process does not consider. In this paper we have assumed both produced items and production equipment deteriorate. The objective of the present paper is to determine the production inventory schedule and maintenance inspection schedules. When the production system deteriorates it shift to an out of control state and begins to produce a proportion of defective items necessitating corrective maintenance action. In this paper we examined the above issues with a perceptive production inventory model for deteriorating items. Manufacturer sells goods to multiple markets with different selling seasons. A model is formulated to integral several realistic aspects, including items, and process deterioration varying demand and production rates, quality, inspection and maintenance. The demand has been assumed exponential decreasing time dependent. Cost component affected by both inflation and time value of money.

A numerical example was illustrated to explain the model and sensitivity analysis has carried out by taking major parameters.

Keyword: EPQ , Deterioration, Inflation, Time dependent demand

1. Introduction

In many inventory models it is assumed that decay and deterioration is very important for the inventory model. Certain types of items undergo changes while in storage so that, with time, they become partially or entirely unfit for use. Deterioration refers to damage, spoilage, vaporization, or obsolescence of the products. There are several types of items that will deteriorate if stored for extended periods of time. Examples of deteriorating items include metal parts, which are prone to corrosion and rusting, and food items, which are subject to spoilage and decay. Electronic components and fashion clothing also fall into this category, because they can become obsolete over time and their demand will typically decrease drastically.

Inventory control for deteriorating items has been a well-studied problem.Numerous optimal and heuristic approaches have been developed for modelling and solving different variations of this problem.

The global markets of today offer to the manufacturer more selling opportunities, as well as production location choices. Geographically dispersed provide multiple selling opportunities to the manufacturer. Another important advantage is the opportunity to exploit the different timing of the selling season at the various markets.

Kouvelis and Gutierrez (1997) and **Khouja (2001)** both gave examples from the US garment industry. They found that a US garment manufacturer could sell his summer fashion items to an Australian clothing retailer after US's. In other cases, the difference in the timing of the selling seasons represented a time lag in the phases of the product life cycle of the same brand (or model) in different geographic markets. For example, athletic shoes traditionally have a six-month time lag between the product life cycles of various models in the North American and European markets (New Balance Athletic Shoes, 1980). Sometimes, because of limitations of capacity or capital, the manufacturer will supply to the different market with a different selling season in order to satisfy the demands of all markets. For example, when demand increase suddenly because of political events or natural disasters, the manufacturer often first offers the products to the most important market, then to the secondary market, and so on.

The presence of geographically dispersed markets for deteriorating items creates profitable opportunities in multiple-markets for a single location manufacturer. At the same time, because of the different selling seasons in different markets, the manufacturer's production planning and inventory problems become more complicated. Under the circumstances, the manufacturer will face a fluctuating demand, which is a discontinuous function.

Inspection and maintenance seems to be a genuine need for looking into this aspect in order to make the analysis closer to reality. Past research have not been examined such type of phase of producing the bad quality items .

Thus the issue consider in this paper is incorporate quality, inspection and maintenance, into the multiple market model with deteriorating items and exponential decreasing demand rate.

2. Literature Review

Goyal and Giri (2001) provided the most recent review of the literature on deteriorating inventory models. **Goswami and Chaudhuri (1992)** formulated and analytically solved the inventory replenishment problem for a deteriorating item with linearly time-varying demand, finite shortage cost, and equal replenishment intervals. **Wee (1995)** proposed a replenishment policy for a deteriorating product where demand declines exponentially over a fixed time horizon with constant deterioration and complete or partial backordering. Models were numerically solved and the policies compared assuming that shortages are allowed. **Su and Lin (2001)** optimally solved a production- inventory model for deteriorating products with shortages, in which the demand is exponentially decreasing and the production rate at any time depends on both the demand and the inventory level.

Some researchers **(Balkhi and Benkherouf, 1996; Papachristos and Skouri, 2000)** have presented a method for finding ‘‘the optimal replenishment of inventory” system model for deteriorating items, where demand is allowed to vary with time in an arbitrary way. However, there were also some studies **(Chung et al., 2000; Giri et al., 1996; Giri and Chaudhuri, 1998; Hou, 2006; Wu et al., 2006)** which have discussed inventory models for deteriorating items where the demand rate is a function of the on-hand inventory. Other authors such as **Covert and Philip (1973), Goswami and Chaudhuri (1991), Manna and Chaudhuri (2006) and Chang et al. (2006)** developed inventory models for deteriorating items with time-dependent demand. The common characteristic of the above papers was that the demand function is continuous and differentiable.

Yang and Wee (2002) developed an integrated economic ordering policy for deteriorating items for a single vendor and multiple buyers. However, in this model, all buyers were assumed to be operating in the same market and for the manufacturer; the demand rate was a constant. However, with increasing global co-operation and competition, more and more manufacturers sell their products to different markets

World wide. Since the each market has a different selling season, the demand function will be not continuous. In fact, the demand function will become a piecewise function with time. Thus, the traditional inventory models for deteriorating items will be ineffective **Lee and Rosenblatt (1989)** determined the production and inspection schedules for a system in which the cost of maintenance depends on detection delay, i.e., number of periods in the out-of-control state. The optimum maintenance schedule is determined as a function of the cost of defective items, the cost of restoration, and the mean time until the system is out-of-control. The model of **Lee and Rosenblatt (1989)** will constitute a major element of our model. **Posner et al. (1994)** analysed a number of producing machines that are either stopped for repair when they fail naturally, or stopped deliberately due to capacity limitations or other considerations. **Iravani and Duenyas (2002)** formulated a Markov decision process to jointly plan production and maintenance of a single deteriorating Machine with an increasing failure rate to satisfy a stochastic demand. In their model, machine deterioration is assumed to decrease its production rate and increase the time and cost of its maintenance.

Quality control was integrated with production/inventory control and maintenance scheduling in several models. **Rahim (1994)** presented a model to simultaneously determine the production lot size, inspection schedule, and control chart parameters for a deteriorating production process with a general time-to-failure distribution and increasing failure rate. The objective is to minimize the expected total cost of quality control and inventory control per unit time. **Ben-Daya (1999)** extended this work by assuming that the reduction in the age of a deteriorating

system is proportional to the level of preventive maintenance. **Wang and Sheu (2001)** determined the optimal lot size and inspection threshold for deteriorating production equipment, assuming the first group of items of each batch is not inspected.

Rahim and Ben-Daya (2001) provided the only previous model in which the deterioration of both the items produced and the production equipment is considered.

They combined the deteriorating inventory model of Misra (1975) with the deteriorating equipment model of **Rahim (1994)**. The combined model is used to determine the optimal lot size, inspection schedule, and control chart limits, with the assumptions of arbitrary distribution of deterioration times, normal distribution of the quality characteristic, and constant demand and production rates. In comparison, we assume that both the demand rate and the production rate are time dependent, with an exponentially decreasing demand and demand-dependent production

rate. In this paper, the effects of equipment deterioration, variable demand and Production-inventory model for deteriorating items.

production rates, and inspection and maintenance decisions will be included in an integrated model of deteriorating inventory.

3. Model Development

For our model, the following aspects will be incorporated:

1. Considering the possibility of producing bad quality items, given the occurrence of a shift in the process to the out-of-control state.
2. Incorporating both the production-inventory schedule and the inspection maintenance schedule, since the process will not return to the in-control state unless it is inspected and restored.
3. Modelling both deteriorating production equipment as well as deteriorating inventory items.
4. Assuming the production rate to be a function of the demand rate, and both to be varying with time.

4. Assumptions

The following assumptions were used to formulate the problem:

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1. A single product, a single manufacturer and multiple-markets were assumed.
2. Demand rate is known and decreasing with time $d_k(t) = A_k e^{-\lambda_k t}, A_k > 0, \lambda_k > 0$ where A_k is the initial demand rate λ
3. Production rate is deterministic and demand dependent.
   $p(t) = a + bd_k(t), a > 0, 0 < b < 1,$
4. Lead time was assumed to be negligible.
5. Deterioration rates of the materials and finished products are deterministic and constant.
6. Shortages are not allowed.
7. Inflation considered for all costs during production unit
8. Time horizon is finite.
9. There is no repair or replacement of deteriorated units during the planning horizon.
10. Items which are defective or have deteriorated are neither repaired nor replaced.
11. At the start of the production cycle the system is in the in-control state i.e., it does not produce any defective items.
12. The elapsed time before the process shifts to the out-of-control state is an exponentially distributed random variable with mean $1/\mu$.
13. During the out-of-control state, α of the total items produced will be defective.
14. Once a shift has occurred, the process will stay in the out-of-control state unless discovered (by inspection) and restored.
15. The scheduled maintenance inspections are performed at equal intervals.
16. The cost of restoration is a function of the duration of the out-of-control state, i.e., of τ .
17. Scheduled inspections and restorations are error-free and instantaneous**.**

5. Notations

Let the parameters of manufacturer and raw material costs incurred by the manufacturer
be as follows:

c_s	Set up cost
p	Production rate
c_p	Unit production cost of deteriorating items
h_p	Unit holding cost of finished products per unit
θ	Constant deterioration rate of finished products.
$I_i(t)$	Inventory level in the ith interval ($i = 1,2,3...m-1$)
$I_j(t)$	Inventory level in the jth interval ($j = m, m+1, ...l$)
α	Fraction of bad items out-of-control state, $0 < \alpha < 1$
$d_k(t)$	Demand rate at time $t, k = 1,2,3...., m-1, m.....l$
$E(N)$	Expected number of defective items produced per unit time During the production interval
TC	Total average cost of the system per unit time
k_d	Deterioration cost per unit
k_t	Cost of each inspection
k_q	Cost resulting from producing a defective item
$1/\mu$	Mean time until the shift to the out-of-control state
n	Total number of inspections per cycle, all performed in the Interval
τ	Detection delay, i.e., the elapsed time from the shift to the Out-of-control state to the inspection/restoration time
s_r	Ordering cost of raw material
c_p	Unit price of raw materials
h_p	Holding cost of raw materials per unit time for the Manufacturer
f	Unit usage of raw materials per finished product
θ_r	Deterioration rate of raw materials
$I_r(t)$	Inventory level of raw materials
q_r	Lot-size per delivery from supplier to manufacturer
n_r	Number of raw material' supplier to Manufacturer
r	Inflation and time value of money.
$R(\tau)$	Restoration cost as a linear function of detection delay τ

6 .MATHEMATICAL MODEL

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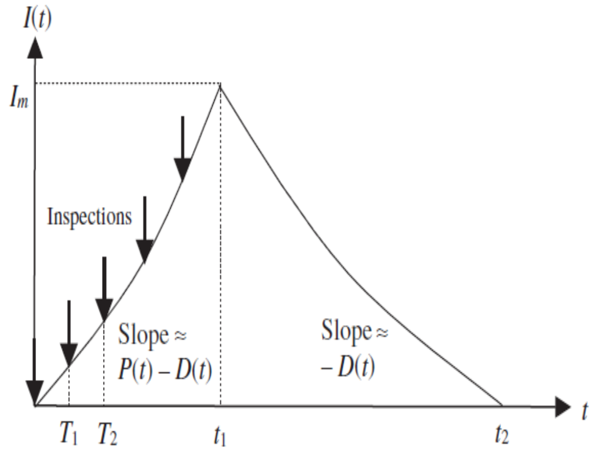

Figure 1. The production-inventory cycle.

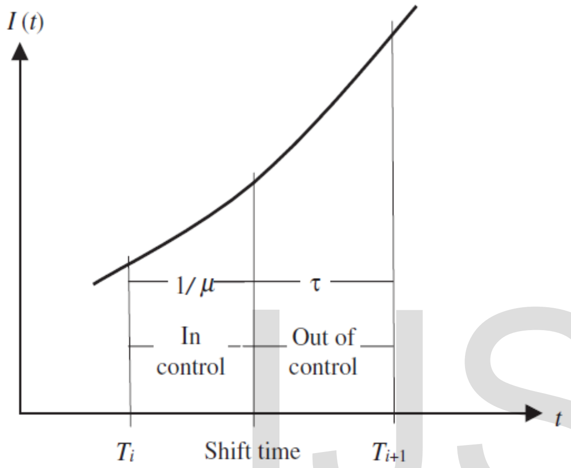

Figure 2. Process shift during a production cycle.

6.1 The inventory equations

The methodology adopted in this paper involves a number of steps. First, the differential inventory equations for all the periods are developed. These Differential equations are solved to formulate the cost model. Finally, a heuristic numerical algorithm is proposed to search for the minimum-cost solution. The details of this methodology are discussed below.

In order to develop the differential inventory equations, we need to define the two stages of the production-inventory cycle shown in figure 1,

A simplified representation of the production cycle has been given below. The effects of deterioration and the production of defective-items are ignored in figure 1 .

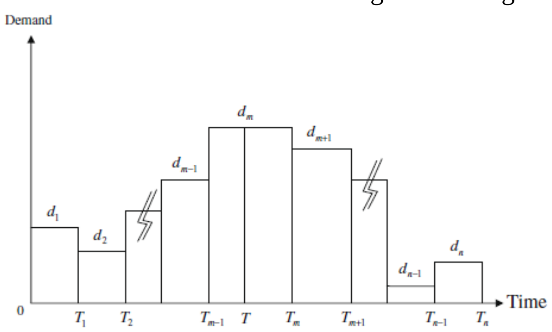

Fig. 1. Time versus demand.

The two stages are the production period $t \in [0,T]$ and the consumption period $t \in [T, T_l]$.

Production period [0, T]. During this stage, the inventory of good items increases due to production but decreases due to demand, deterioration, and the production of defective items. Thus, the inventory differential equation (IDE) is

$$\frac{dI_k^{+}}{dt} + \theta I_k = p - d_k(t) - E(N)$$

$$T_{k-1} < t < T_k \qquad (1)$$

$$k = 1,2,3.........m-1$$

$$\frac{dI_m^{-}}{dt} + \theta I_m = p - d_m(t) - E(N)$$

$$T_{m-1} < t < T \qquad (2)$$

The expected number of defective items produced per unit of time is given by

$$E(N) = \frac{1}{T}\sum_{i=0}^{n-1}\int_{T_i}^{T_{i+1}} \alpha p(t)(T_{i+1} - T_i - t)\mu e^{-\mu t}\,dt$$

$$(T_{i+1} - T_i) = T/n$$

$$E(N) = a\alpha[T + \frac{n}{\mu}e^{-\mu t} - \frac{n}{\mu}] + \text{......}$$

$$\frac{b\alpha a\mu}{T}[\frac{T}{\mu+\lambda} + \frac{n}{(\lambda+\mu)^2}e^{-(\lambda+\mu)T/n} + \frac{n}{(\lambda+\mu)^2} \qquad (3)$$

Solving Eq'n (1)

$$I_k(t) = \frac{a}{\theta}(1 - e^{-\theta t}) + \frac{(b-1)A_k}{\theta - \lambda_k}(e^{(\theta-\lambda_k)t} + e^{-\theta t} - e^{-\theta_k t}) +$$

$$\frac{E(N)}{\theta}(e^{-\theta t} - 1) + \frac{b-1}{\theta - \lambda_1}e^{(\theta-\lambda_1)t - \theta t}$$

$$2 < k < m-1 \qquad (4)$$

$$\& \; I_1(t) = \frac{a}{\theta}(1 - e^{-\theta t}) + \frac{(b-1)A_k}{\theta - \lambda_k}(e^{-\lambda_k t} - e^{-\theta t})$$

$$+ \frac{E(N)}{\theta}(e^{-\theta t} - 1) \qquad (5)$$

Putting $k = m$ in eq'n (4) the sol'n of eq'n (2)

$$I_m(t)=\frac{a}{\theta}(1-e^{-\theta t})+\frac{(b-1)A_m}{\theta-\lambda_m}(e^{(\theta-\lambda_m)t}+e^{-\theta t}-e^{-\theta_m t})+\frac{E(N)}{\theta}(e^{-\theta t}-1)+\frac{b-1}{\theta-\lambda_1}e^{(\theta-\lambda_1)t-\theta t} \tag{6}$$

6.2 Consumption period $[T, T_l]$. The IDE during the consumption period, with no production and subsequently no reduction in the inventory level due to production of defective items, is given by

$$\frac{dI_m^{+}}{dt}+\theta I_m^{+}=-d_m(t)$$

$$T\le t\le T_m \tag{7}$$

$$\frac{dI_j}{dt}+\theta I_j=-d_j(t)$$

$$T_{j-1}\le t\le T_j \tag{8}$$

$$j=m+1, m+2.........l-1, l$$

$$I_j(t)=\frac{A_j}{(\lambda_j-\theta)}e^{-\lambda_j t}+\frac{A_{j-1}}{(\lambda_{j-1}-\theta)}e^{(\lambda_{j-1}+\theta)T_{j-1}}+\frac{A_l}{(\lambda_l-\theta)}(e^{(\lambda_l+\theta)T_l}-e^{(\lambda_l-1+\theta)T_{l-1}}) \tag{9}$$

Putting $j=m$ in eq'n (9) we get,

$$I_m^{+}(t)=\frac{A_m}{(\lambda_m-\theta)}e^{-\lambda_m t}+\frac{A_{m-1}}{(\lambda_{m-1}-\theta)}e^{(\lambda_{m-1}+\theta)T_{m-1}}+\frac{A_n}{(\lambda_n-\theta)}(e^{(\lambda n+\theta)T_n}-e^{(\lambda n-1+\theta)T_{n-1}}) \tag{10}$$

for $T\le t\le T_l$

The condition $I_m^{-}(t)=I_m^{+}(t)$ yields that

$$\frac{a}{\theta}(1-e^{-\theta T})+\frac{(b-1)A_m}{\theta-\lambda_m}(e^{(\theta-\lambda_m)T}+e^{-\theta T}-e^{-\theta_m T})+\frac{E(N)}{\theta}(e^{-\theta T}-1)+\frac{b-1}{\theta-\lambda_1}e^{(\theta-\lambda_1)T-\theta T}=$$
$$\frac{A_m}{(\lambda_m-\theta)}e^{-\lambda_m T}+\frac{A_{m-1}}{(\lambda_{m-1}-\theta)}e^{(\lambda_{m-1}+\theta)T_{m-1}}+\frac{A_l}{(\lambda_l-\theta)}(e^{(\lambda n+\theta)T_n}-e^{(\lambda n-1+\theta)T_{n-1}}) \tag{11}$$

For solving these above Eq'n for T we can directly calculate value of optimal production time T by knowing the some constants value.

7. Cost Component

The inventory holding cost is applicable for both period production and consumption described by:

7.1 Holding Cost

$$HC=h_p\{\sum_{k=1}^{m-1}\int_{T_k}^{T_{k-1}}I_k(t)e^{-rt}dt+\int_{T_{m-1}}^{T}I_m^{-}(t)e^{-rt}dt+\int_{T}^{T_{m+1}}I_m^{+}(t)e^{-rt}dt+\sum_{j=m+1}^{l-1}\int_{T_j}^{T_{jj+1}}I_k(t)e^{-rt}dt\} \tag{12}$$

$$\sum_{k=1}^{m-1}\int_{T_k}^{T_{k-1}}I_k(t)e^{-rt}dt=$$
$$\frac{a}{\theta}(T_0-T_{m-1})+\frac{1}{(\theta+r)}(e^{-(\theta+r)T_0}-e^{-(\theta+r)T_{m-1}})-\frac{E(N)}{\theta(\theta+r)}(e^{-(\theta+r)T_0}-e^{-(\theta+r)T_{m-1}})+\frac{E(N)}{\theta r}(e^{-rT_0}-e^{-rT_{m-1}})-$$
$$\frac{(b-1)A_1}{(\theta-\lambda_1)r}e^{(\theta-\lambda_1)T_1}(e^{-rT_0}-e^{-rT_{m-1}})+\frac{(b-1)A_1}{(\theta-\lambda_1)+(\theta+r)}(e^{-(\theta+r)T_0}-e^{-(\theta+r)T_{m-1}})$$

$$where,\ j=T, m+1, m+2...........n)$$

$$HC = h_p[\frac{a}{\theta}\{T_o + T - 2e^{-rT_{m-1}}) + \frac{1}{(\theta+r)}(e^{-(\theta+r)T_O} + e^{-(\theta+r)T} - 2e^{-(\theta+\bar{r})T_{m-1}})$$

$$+\frac{E(N)}{\theta}\{\frac{1}{r}(e^{-(r)T_O} + e^{-(r)T} - 2e^{-(\theta+r)T_{m-1}})$$

$$-\frac{1}{(\theta+r)}(e^{-(\theta+r)T_O} + e^{-(\theta+r)T} - 2e^{-(\theta+r)T_{m-1}})\} + \frac{(b-1)}{(\theta-\lambda_1)}\{\frac{1}{(\theta+r)}$$

$$(e^{-(\theta+r)T_O} + e^{-(\theta+r)T} - 2e^{-(\theta+r)T_{m-1}}) - \frac{e^{-(\theta-\lambda_1)}}{r}(e^{-(r)T_O} + e^{-(r)T} - 2e^{-rT_{m-1}})$$

$$-\sum_{k=1}^{T}\frac{(b-1)A_k}{(\theta-\lambda_k)}[\frac{1}{(\theta+r)}(e^{-(\lambda_k+r)T_{k-1}} + e^{(\theta-\lambda_k)T_{k-1}-(\theta+r)T_k})$$

$$+\frac{1}{(\theta+r)}(e^{-(\theta+r)T_{k-1}} - e^{-(\theta+r)T_k}) + \frac{1}{(\lambda_k+r)}(e^{-(\lambda_k+r)T_{k-1}} - e^{-(\lambda_k+r)T_k})$$

$$-\frac{A_n}{r(\lambda_n-\theta)}(e^{(\lambda_n+\theta)T_n} - e^{(\lambda_n+\theta)T_{n-1}})((e^{-(r)Tn} - e^{-(r)Tm-1} + e^{-(r)Tm+1} - e^{-(r)T})$$

$$+\sum_{j=T}^{l}\frac{A_j}{(\lambda_j-\theta)(\lambda_j+r)}(e^{-(\lambda_j+r)T_j} - e^{-(\lambda_j+r)T_{j-1}}) -$$

$$\frac{A_{j-1}}{(\lambda_{j-1}-\theta)r}(e^{-(\lambda_{j-1}+\theta)T_{j-1}-rT_j} - e^{-(\lambda_{j-1}+\theta)T_{j-1}-rT_{j-1}}),$$

$$\ldots(13)$$

7.2 Deterioration Cost

The deterioration cost of the finished product

$$DC = k_d\theta\{\sum_{k=1}^{m-1}\int_{T_k}^{T_{k-1}} I_k(t)e^{-rt}dt + \int_{T_{m-1}}^{T} I_m^{-}(t)e^{-rt}dt + \int_{T}^{T_{m+!}} I_m^{+}(t)e^{-rt}dt + \sum_{j=m+1}^{l-1}\int_{T_j}^{T_{j+1}} I_k(t)e^{-rt}dt \quad (14)$$

7.3 Setup Cost

The setup cost

$$SC = c_s \quad (15)$$

7.4 Inspection Cost

A total n inspection are carried out during the production period stage $[0, T]$ of each cycle for a fixed cost k_t per inspection.
The total inspection cost per unit

$$C_i = nk_t \quad (16)$$

7.5 Quality Cost

Quality cost is the cost incurred due to the production of defective items during the out of control phase of the process.

$$C_q = k_q \int_0^T E(N)e^{-rt}dt$$

$$C_q = \frac{k_q}{r}E(N)(1-e^{-rT}) \quad (17)$$

7.6 Restoration Cost

$$C_R = [\sum_{i=0}^{n-1}\int_{T_i}^{T_{i+1}} \mu R(\tau)e^{-\mu(T_{i+1}-T_i-\tau)}e^{-r\tau}d\tau$$

$$R(\tau) = 10 + .15\tau$$

$$C_R = n\mu e^{-\mu T/n}[-10\frac{(e^{-\mu T/n}-1)}{-\mu} +$$

$$0.15(T/n\frac{e^{T/n}}{\mu} - \frac{(e^{\mu T/n}-1)}{\mu^2}] \quad \mathbf{(18)}$$

6.4 The total cost of the manufacturer inventory level

$$TC = HC + DC + SC + C_i + C_q + C_R$$

8. Manufacturer warehouse raw material inventory model

The inventory system of raw materials is illustrated in Fig. 2. The replenishment lot-size

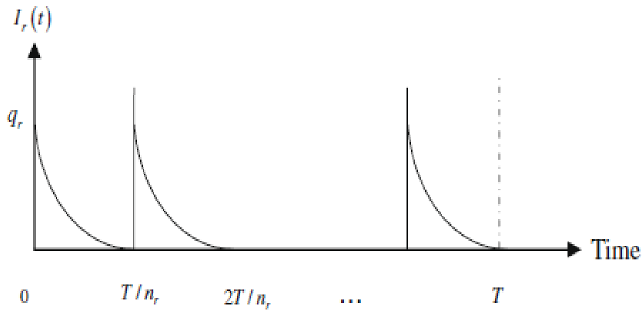

Fig. 2. Raw material's inventory system.

is replenished at $t = 0$. During the period T/n_r, the inventory levels of raw materials decrease due to demand and deterioration until they are zero at $t = T/n_r$. The instantaneous raw materials inventory level at time t can therefore be represented by the following differential equation:

$$\frac{dI_r(t)}{dt} + \theta_r I_r(t) = -f$$

$$0 \le t \le T/n \quad (19)$$

Using the boundary condition $I_r(t) = 0, t = T/n_r$

$$I_r = (a_p + b_p T/n_r)\frac{1}{\theta_r}e^{\theta_r}T/n_r - (a_p + b_p t)\frac{1}{\theta_r}e^{\theta_r t} - \frac{b_p}{\theta_r^{\,2}}(e^{\theta_r T/n} - e^{\theta_r t}) \tag{20}$$

The holding cost of the raw materials

$$TH_r = n_r h_r \int_0^{T/n_r} I_r(t)e^{-rt}dt$$

$$TH_r = n_r h_r[\frac{1}{r}(1-e^{-rT/n_r}) + \frac{1}{\theta_r}\{\frac{1}{(\theta_r - r)}(a_p + b_p T/n_r)(e^{(\theta_r - r)T/n_r} - a_p) - \frac{1}{r^2}(e^{(\theta_r - r)T/n_r} - 1) - \frac{b_p}{r\theta_r^{\,2}}e^{\theta_r T/n_r}(1-e^{-rT/n_r}) + \frac{1}{(\theta_r - r)}(e^{(\theta_r - r)T/n_t} - 1)] \tag{21}$$

The deterioration cost of the raw materials is

$$TD_r = c_p \theta_r n_r [q_r - \{\frac{-1}{r}(a_p + b_p T/n_r)e^{-rT/n_r} + \frac{b_p}{r} - \frac{b_p}{r^2}(e^{-rT/n_r} - 1)\}] \tag{22}$$

The setup cost of the raw material is

$$TS_r = n_r s_r \tag{23}$$

The total cost of raw materials,

$$TC_r = TD_r + TH_r + TS_r \tag{24}$$

The integrated total cost of raw materials &finished products TC is the sum of all individuals cost defined by,

$$TC = HC + DC + SC + C_q + C_R + C_i + TD_r + TH_r + TS_r \tag{25}$$

To minimize this function we employ a heuristic numerical solution algorithm to determine the value of n_r for the correspond number of inspection n

The optimize solution can be obtained by,

$$\frac{\partial TC}{\partial n_r} = 0 \;\; \& \;\; \frac{\partial^2 TC}{\partial n_r^{\,2}} > 0$$

9. Algoritm Description

In order to minimize TC the algorithm goes through the following steps:

1. Since 'n' is the number of inspection per period T_n start from $n \geq 1$.
2. Find the first derivative of TC $w.r$ to n_r $s.t$ $\frac{\partial TC}{\partial n_r} = 0$. Solve this $eq'n$ for n_r for each value of n denoted by $n_r(n)$
3. Putting these value in $eq'n$(25) And defined TC for each n.
4. Repeat step (2) &(3) for all possible value of n until the minimum .TC is found. The optimum value of n & the optimum sol'n that satisfies the following condition:
   $TC(n_r(n^*-1),(n^*-1)) \geq$
   $TC(n_r(n^*),(n^*)) \leq TC(n_r(n^*+1),(n^*+1))$
5. It can be shown that the integrated cost which is the optimum value is $TC(n_r(n^*),(n^*))$
6. Find the optimum solution of the T ,q_r, n_r from $eq'n$(11),(12)

10. Numerical Example

The following parameter is used to find the optimum cost

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$\theta = 0.01/u,$

$\theta_r = 0.03/u,$

$f = 1.0,$

$h_p = 0.15$

$A_1 = 100, A_2 = 120,$

$\lambda_1 = 0.2, \lambda_2 = 0.1$

$a = 500, b = 0.5$

$T_1 = 1 \to 20w, T_2 = 8 \to 30w,$

$s_r = 100/cycle, k_s = 150/cycle, c = 10, c_r = 3,$

$h_r = 0.1, r = 0.01$

$\alpha = 0.2, \mu = 0.1, k_t = 3/w, k_q = 4, R(\tau) == 10 + 0.15\tau,$

n	n_r	q_r	$TC_p(10^{-4})$	$TC_r(10^{-4})$	$TC\,(10^{-4})$
1	4	1011.2	14342	14586	35300
2	5	2334.6	18742	18950	25192
3	6	1945.3	11232	10892	25321
4	6	2221.3	23903	20883	35242

11. Convexity

By applying the solution procedure as given in algorithm we can get the convexity at

n =3, n_r =6 , q_r =19453, TC =25321

12. Sensitivity Analysis

The change in the value of parameter may happen due to uncertainties in any decision making situation. In order to examine the implication of these changes, a sensitivity analyse is will be of great help in decision making.

We performed the sensitivity analysis on the optimum solution of the model with respect to major parameter such as $p, d_i, \theta, \theta_r, \alpha, \mu, \lambda, k_t, k_q$

The result shown in Table (2)

Effect of change in various parameters of the production inventory model:

At n =3, n_r =4

Input	% chg.	q_r	T	TC_r	TC_p	TC
p	10 -10	1.01 - 10.24	-5.01 7.23	-4.61 2.04	10.11 -14.30	5.44 -11.45
d_i	10 -10	12.10 21.11	4.23 -3.31	3.21 3.04	-9.34 7.34	3.11 6.03
θ	10 -10	11.31 -5.06	5.32 -5.11	3.74 -4.56	8.34 7.17	1.12 0.03
θ_r	10 -10	8.16 -7.65	2.43 -3.42	-4.11 -7.23	2.433 -2.013	2.433 -2.012
f	10 -10	7.34 -5.87	5.32 -522	3.21 1.21	0.1234 -5.012	0.124 -3.244
α	10 -10	3.11 2.37	10.12 4.23	17.14 15.31	12.892 5.567	10.562 7.345
μ	10 -10	1.90 -1.74	5.60 2.7	1.929 -2.39	3.654 -2.45	6.467 - 2.3455
λ	10 -10	6.66 2.84	7.94 1.09	3.453 - 3.941	2.4959 3.9559	2.9404 -1.346
k_t	10 -10	3.93 3.12	4.36 -2.44	1.234 - 0.922	5.9696 - 2.5466	3.5860 2.5960
k_q	10 -10	2.22 4.67	3.23 -0.87	2.455 2.290	2.4040 - 3.2032	4.9585 2.4994
$R(\tau)$	10 -10	5.84 3.43	3.56 2.34	1.263 - 1.209	4.5676 - 1.2345	4.6895 2.4949

1. The total cost was negatively correlated with all other parameter.
2. The raw material cost are least sensitive with the parameter α and more sensitive by the μ
3. The value of the production period is most sensitive to k_t, k_q and the least sensitive to $R(\tau)$
4. When the production rate p decreases q_r, TC_p, TC will also decrease . But this trend is reversed for the optimal solution TC_r , T . Intuitively, improving the production rate can increase the manufacturer' profit.
5. When all $d_i = 1,2..$ decrease, TC_r , T tend to decrease while q_r, TC_p, TC tend to increase.
6. TC_r , T TC_p tend to decrease with decrease s parameter θ .When θ varies, TC and TC_p are less sensitive but and TC_r , T are more sensitive. The main reason is that if θ decreases, the deterioration quantity of finished products will decrease. Therefore, the manufacturer can reduce their production quantity of finished products and the demand of raw materials. Accordingly, this will decrease the costs of finished products and raw materials. Hence, trying to find a new technique to decreasing θ is an effective way to decrease the total cost TC .
7. As θ_r and f decreases the changes in TC_r and TC is more sensitive.

13. Conclusion

In this paper we have suggested a method for finding the production and inventory schedule of the deteriorating items .In general we presented a model and a solution

algorithm for incorporating quality and maintenance aspects into production inventory system. Closed form solutions are rare in models that include the quality considerations. Exploiting the difference in timing of the selling season of the deteriorating items at different markets is a unique opportunity to improve the profitability .Here, we assumed the manufacturer produces in one location and sells in different markets that have different selling seasons. We have represented that our method helps minimize costs.
In the proposed model the demand rate in each market is exponential decreasing function with time. Production rate is dependent on demand. Further research could result in model that is representative of many practical situations that is useful as a decision making tool for management.

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